White Paper

# Implied Volatility Surface

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*Originally published 19-Aug-2015. Updated 24-Jan-2017.*

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## Introduction

FactSet's Implied Volatility Surface calculation methodology aims to provide an accurate parameterized representation of market implied volatility (IV) skews, allowing continuous results to be calculated for a theoretical option with any strike or maturity (see Appendix A for an example). A Vega-weighted second-order polynomial is calculated for each maturity and anchored to the market at-the-money (ATM) IV. Volatility surfaces are calculated daily for all U.S. and international equities, exchange-traded funds (ETFs), and indices that FactSet has options data for (i.e., over 4,000 securities in the U.S. and 1,800 non-U.S.).

The following methodology is used to calculate the surface:

1. Fetch data and apply filters
2. Calculate ATM IV for each maturity
3. Connect put/call skews
4. Estimate regression parameters

To calculate a volatility surface for a security, at least 30% of the maturities must have an ATM IV value that can be calculated. If ATM IV cannot be calculated for over 30% of the maturities, a surface will not be calculated for the security on that day. If ATM IV can be calculated, but less than 30% of the maturities have at least five securities, a sector skew will be applied (see Appendix B for more detail).

## Fetch Data and Apply Filters

The first step in the volatility surface calculation methodology is to fetch the security's option chain and calculate each option's IV.  For American options, this is calculated using the Cox-Ross-Rubinstein Binomial Option Pricing Model (BOPM) with a 50-step binomial tree.  For European options, this is calculated using the Black-Scholes pricing model.  Each option is then passed through several filters to assure it is suitable.  If the option fails at any of these filters, it is removed from the universe and excluded from the following calculations.  The filters that apply to all securities are:

- No options that are in-the-money
- No options that are adjusted
- No options where an IV can't be calculated
- No options on a secondary exchange

The filtered data is then separated into arrays by maturity.  For a complete filter breakdown, see Appendix F.

## Calculate ATM IV and Connect Put/Call Skews

Once the filtered data is arranged into arrays, an ATM IV is calculated for each maturity.  The ATM strike is determined using the forward price instead of the spot price, or it uses the futures price for volatility contracts.  Discrete dividends are projected and used when calculating the forward price for equity securities on the USA and MOD exchanges, and dividend yield is used for index securities and equity securities on international exchanges. For details on the dividend methodologies used when calculating the forward price, see Appendix C.

A straddle is found where the strike price is closest to the forward price, and the put and call IV are averaged to provide an ATM IV for the maturity.

Some maturities may lack sufficient data to calculate an ATM IV.  In this case, an interpolation is performed from the nearest maturities to approximate the value.  If the missing ATM IV is at either end of the maturity array, a flat extrapolation is used by taking the IV from the closest available pole.  For complete details on these calculations, see Appendix D.

When combining puts and calls into a single skew, some securities exhibit a discontinuity for IVs near ATM.  In order to close the gap, an adjustment factor is calculated and applied to the empirical IV data.  The adjustment factor is calculated as ATM put IV – the average of put and call IV.  All options with moneyness less than one are moved "down" by subtracting the adjustment factor from the raw IV.  Similarly, all options with moneyness greater than one are moved "up" by adding the adjustment factor to the raw IV, thereby removing the gap from the data.  For an example showing why the adjustment factor is needed, see Appendix E.

## Estimate Regression Parameters

A Vega-weighted regression is calculated for each maturity to fit the filtered data to a parabola. Each regression is of the form:

$$y = ax^2 + bx + c$$

where:

- $y = \sigma^2(K)T$, variance, annualized for strike K, at maturity T
- $x = \ln\left(\frac{K}{F}\right)$, the log moneyness where F is the forward price for the associated maturity

The values are weighted using the standard Vega calculation.

The regression is calculated using least squares and provides an estimate for the coefficients $a$ and $b$, and intercept $c$. Each intercept $c$ is then replaced with the normalized ATM IV, anchoring the parabola's minimum value so the 100% moneyness theoretical IV will align with the market. This process is repeated for each maturity.

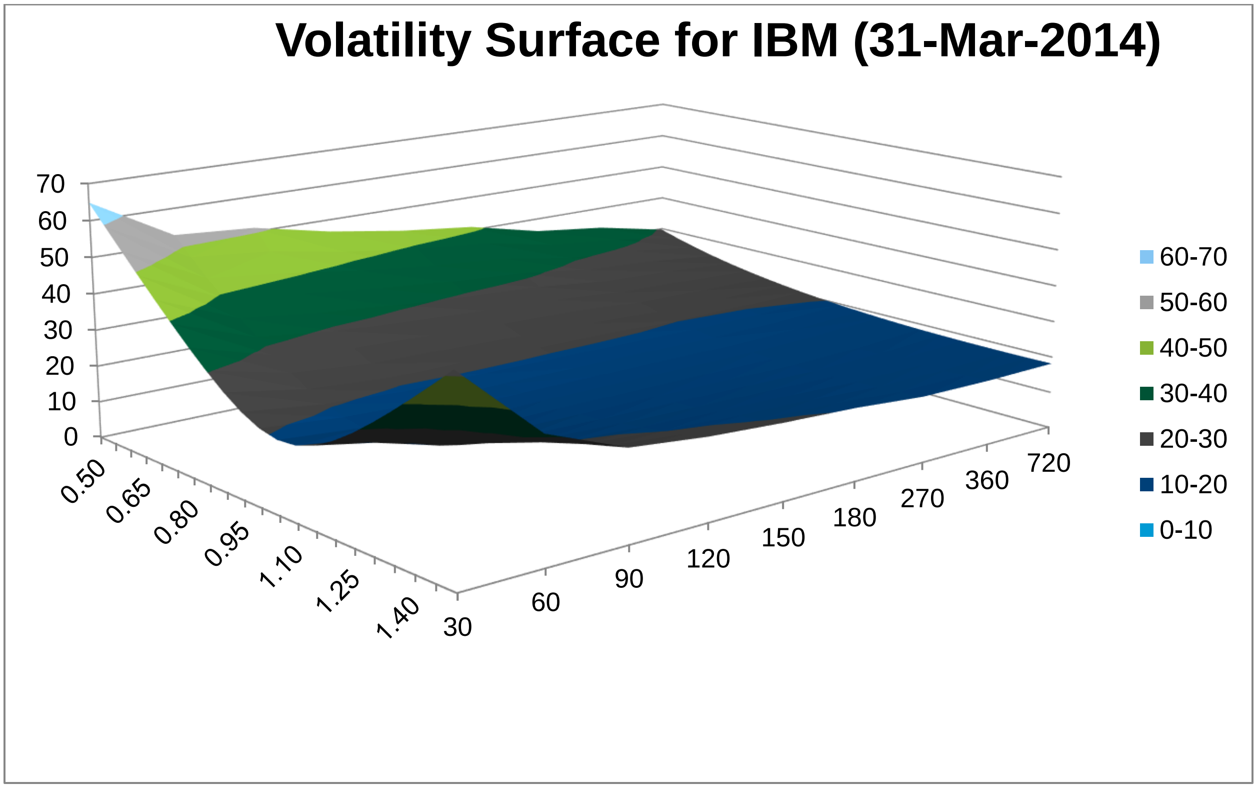
The coefficients are then stored in the database and used to calculate the surface results.

## Appendix A

### Volatility Surface Chart

The following chart is the volatility surface for IBM on 31-Mar-2014. The original option chain fetch returned 909 options, which reduced to 304 after filtering. These 304 options were separated into arrays by maturity. The market maturities in this case were 4, 11, 19, 25, 32, 39, 47, 82, 110, 201, 292, and 655 days to expiration (DTE).

A forward price is calculated for each maturity and then used to find the ATM IV. After running regressions, the IVs are calculated over the range and charted:



## Appendix B

### Sector Composite Skews

In order to calculate a volatility surface for the day, securities must meet two primary requirements:

1. At least 30% of maturities can calculate an ATM straddle
2. At least 30% of maturities have more than five options after filtering

If a security fails the first requirement, the surface is not calculated for that day. If a security passes the first requirement and fails the second, a sector composite skew will be overlaid onto the security's ATM points. Sector composites are only applied to securities on the USA exchange. In addition, the sector composite is only applied for securities that fail the second requirement on a regular basis. Once a security is flagged to use the sector composite, it will continue to do so. This prevents inconsistent results from occurring due to the sector composite being applied on some days and not others. Securities on non-U.S. exchanges will return NA if they fail the second requirement.

Sector composite skews are calculated daily for each of the 20 FactSet economic sectors. All securities in each sector where a volatility surface can be calculated contribute to the sector composite skew.

To calculate the sector composite skew, implied volatilities are calculated for each security for 50% to 150% moneyness at 10% intervals at 30, 60, 90, 120, 150, 180, 270, 360, and 720 DTE. A market-cap weighted average IV is calculated for each moneyness and DTE combination. A second-order polynomial regression is then calculated on the moneyness values, the coefficients of which make up the sector composite skew.

When applying the sector skew to a security, the skew coefficients are transformed to align with the security's DTEs. The underlying security's ATM point at each maturity is used as the intercept while the transformed sector composite coefficients determine the shape of the skew.

# Appendix C

## Dividend Methodologies

The dividend methodology used when calculating the forward price is determined by both the option type and the data available on the underlying ID. For Equity Options, discrete dividends will be used. For index options, a continuous dividend yield will be used.

### Discrete Dividends

Discrete dividends are projected out to the furthest available expiration.

For securities in the U.S. and Canada, the most recent dividend determines the amount and frequency (i.e., quarterly, semi-annual, or annual) of the projected dividends. Any special dividends are ignored and not used in projections. For example, IBM has a dividend of $1.3 with an ex-date of 6-Nov-2015, and pays dividends on a quarterly interval. If no future dividends are announced, the projections will assume the dividend of $1.3 remains constant and continues to be paid quarterly. When calculating the forward price for the 15-Jul-2016 maturity (assuming no new dividend announcements), the projected dividends are: $1.3 on 5-Feb 2016 and $1.3 on - 6-May-2016.

International equities are handled in a manner similar to U.S. and Canada, except projections are based off the past year of dividends rather than the most recent. This is done so that interim and final dividends can be included. Once again, any special dividends are excluded from projections. Using Vodafone as an example, there was an interim dividend of £0.0368 with an ex-date of 19-Nov-2015 and a final dividend of £0.0762 with an ex-date of 11-Jun-2015. Interim and final dividends are projected independently and each occurred once during the year. When calculating the forward price for the 17-Feb-2017 maturity (assuming no new dividend announcements), the projected dividends are:  £0.0762 on 10-Jun- 2016 and £0.0368 on 18Nov-2016.

If the equity does not have a known dividend frequency or discrete dividend information is not available, dividend yield will be used. In these cases the same dividend yield will apply to all maturities.

### Continuous Dividends

For indices, a continuous dividend yield is applied. Indices on exchanges without settlements (USA, BSP, OMX, OSL, and MOD) the same dividend yield will apply to all maturities. When settlement prices are available, an implied dividend yield will be calculated for each maturity.

The implied dividend yield is calculated for each put-call option pair in the filtered universe. The implied dividend yield is calculated as:

$$q_{imp} = \frac{ln\left(\frac{Call\ Settle - Put\ Settle + Ke^{(-rt)}}{s}\right)}{t}$$

Once the implied dividend yield is calculated for each put-call pair, the average value is calculated for each maturity and used when calculating the forward price for that maturity. In the case where the average implied dividend yield is less than zero, no dividend yield is used.

## Appendix D

### Interpolation and Extrapolation

Linear interpolation is used throughout the volatility surface calculation to fill ATM IV holes and convert market maturity skews.

Each skew is parameterized by coefficients A, B, and C. When a point for a DTE that is not a market maturity is requested, the two closest market skews surrounding that DTE are found. The A, B, and C from each of the surrounding skews are interpolated to calculate an A, B, and C for the requested maturity. The interpolated coefficients are then used to calculate IVs for that maturity.

If only one pole is available, the IV is determined using extrapolation. This occurs when calculating values for DTEs beyond either end of the security's maturity spectrum. For the requested DTE, the A coefficient from the nearest pole is re-used. The new B coefficient is calculated using:

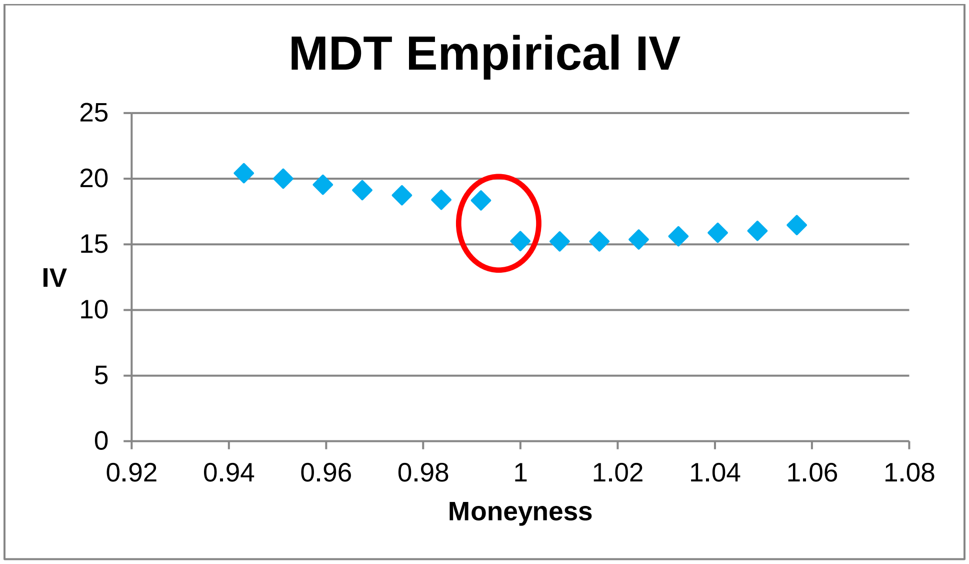
$$New\ B = Nearest\ DTE\ B\ Coefficient * \frac{New\ DTE^{0.5}}{Nearest\ DTE^{0.5}}$$

For the intercept, the nearest C coefficient is copied and re-annualized with the new DTE value.

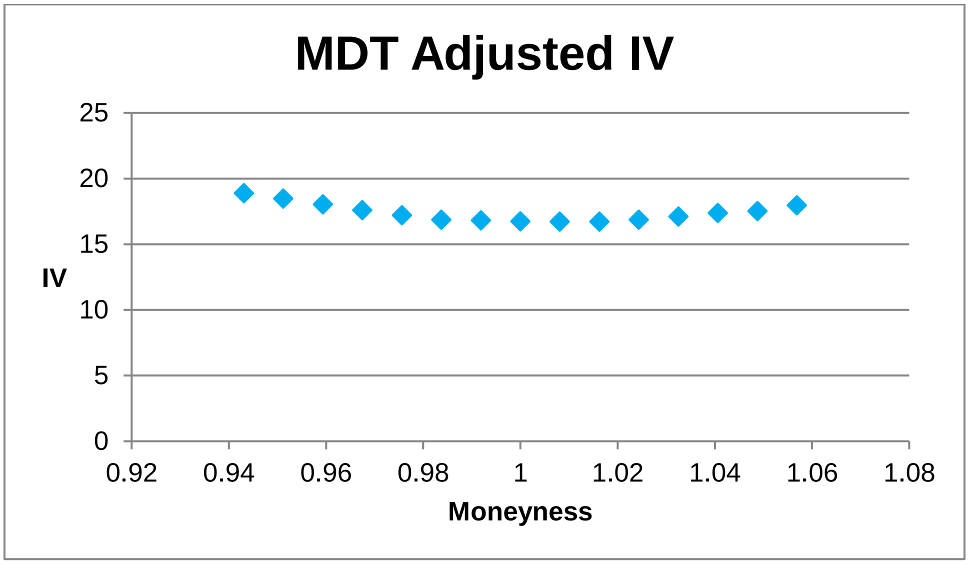
## Appendix E

### Adjustment Factor

One of the first steps in calculating the volatility surface is adjusting empirical IV data to remove any gaps when the put and call skews are combined. You can clearly see this gap in the market data for Medtronic (MDT) on 31-Mar-2014 with 25 DTE. In the first chart, a gap is clearly seen at moneyness = 1:




The adjustment factor removes this gap and provides a smoother curve, allowing for more accurate regression parameters:



# Appendix F

## Filtering Rules

The following filtering rules are applied to the following exchanges (both equity and index securities):

| | ASX | BOM | BSP | DMI | EUR | KRX | LIF | MOD | MRV | OMX | OSE | OSL | TFEX | USA | WAR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No options that are in-the-money | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| No options that are adjusted | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| No options where an IV can't be calculated (NA or Zero value) | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| No options on a secondary exchange (Primary exchange only) | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| Check if the option type is equity or index. If equity, filter out any European style option | X | | X | X | X | X | X | X | X | X | X | X | X | X | X |
| No options with bid or ask equal to 0 | | | X | | | | | X | | X | | X | | X | |
| No options where the last settlement date is not equal to requested date | X | X | | X | X | X | X | | X | | X | | X | | X |
| No options with a settlement price equal to NA or 0 | X | X | | X | X | X | X | | X | | X | | X | | X |
| No options with contract size not equal to 100 | | | | | | | | | | | | | | X | |
| No options where the bid/ask percentage spread is less than 5% | | | | | | | | | | | | | | X | |
| For each underlying ID, calculate the Contract Size's mode. Filter out all options with a contract size not equal to mode | X | X | X | X | X | X | X | X | X | X | X | X | X | | X |